Compensating for the Coriolis Force and Air Resistance in the Videogame Call of Duty 4: Modern Warfare

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Abstract

In this paper we analyze the motion of a bullet's flight path in the popular videogame Call of Duty 4: Modern Warfare. One level in particular challenges players to compensate for the Coriolis force on the bullet as well as wind resistance. We determine the amount by which one would have to compensate for the rotation of the Earth in this fictional environment which is on the order of 10 centimeters. We also take into account quadratic air resistance, showing a clear discrepancy between what is predicted by theory and what is observed in the game itself. We present a number of factors of why this discrepancy occurs, the principle being entertainment value for the game.

1 Introduction

Videogames have been offering more and more realistic physics since their inception. One game in particular, Call of Duty 4: Modern Warfare (COD4), presents an interesting problem of physical analysis. The game places you as a sniper whose job is to assassinate a nuclear arms dealer in the midsts of the post-distaster Chernobyl area. Your adviser tells you that, because of the distance involved, you need to compensate for both wind resistance and the Coriolis force, the force due to the rotation of the Earth on fast moving bodies in the atmosphere. Failing to compensate for the arcing of the bullet due to these two forces will result in alerting the terrorist and his cohorts, effectively failing the mission.

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2 The Coriolis Force

It is well known that the Coriolis force is:

$$F_{cor} = 2mv \times \Omega \Rightarrow F_{cor} = 2mv\Omega\widehat{r} \tag{1}$$

The game displays a distance of 896.7 meters and a travel time of 1.05 seconds[4], which corresponds to a velocity of ≈854 m/s. The mass of the slug corresponding to this velocity is 0.052 Kg. Solving for F_{cor} with Ω as 7.3x10^{-}5 sec^{-1} gives equation 1 as 6.48x10^{-5} N.[3] We can now solve for the acceleration due to the Coriolis force on the slug by setting equation 1 equal to the general acceleration of the slug in its flight path:

$$2\mathrm{m}\dot{x}\Omega = m\ddot{x} \tag{2}$$

Solving for $\ddot{x}$ yields an acceleration of .12 m/s^2.[1] We can now solve for how far we must aim off target to compensate for this acceleration. We start with one of the principle equations of kinematics:

$$d = x_0 + v_0 t + \frac{1}{2}\ddot{x}t^2 \tag{3}$$

Solving for d, gives a compensation distance of 0.07 m, or 7 centimeters.

3 Air Resistance

3.1 Compensation Without Air Resistance

As indicated by a little waving flag throughout this segment of the game, there is a substantial breeze where the target is. Yet, at one particular point, the breeze dies down and the shot can be taken, compensating only for the 7 centimeter offset. Because the target is in the northern hemisphere, the Coriolis force would want to push the bullet to the left, so aiming to the right of the target is key. Quantifying 7 centimeters is on the order of the typical width of a human arm at the bicep, so an approximation of 10 centimeters would work as well, since the ratio of a typical arm width of 10 centimeters to a typical torso width of 60 centimeters is 1/6, it is negligible for the offset of the dead-center hit on the target's torso.

3.2 Compensation With Air Resistance

However, if we wished to fire the bullet at any given point in time, we would have to account for quadratic air resistance. The initial speed of the bullet is very high, and the mass also very high (as far as bullets go), so we can neglect the linear resistive term of air resistance and focus on its quadratic counterpart:

$$m\ddot{x} = -c\dot{x}^2 \tag{4}$$

Now we must compensate for the Coriolis force, which is forcing the bullet to the left, and the force due to the resistance of wind, which is pushing the bullet to the left.[1] Our equation of motion for the x dimension where the target is now:

$$m\ddot{x} = 2m\dot{x}\Omega - c\dot{x_w}^2 \tag{5}$$

Quantifying the speed of the wind in the game is difficult, but since we can see that the flag on the car is flapping upright and, to some extent, rather constantly, we can assume a decent breeze. A strong breeze as defined by the Beaufort scale is, on average, 12.35 m/s. Utilizing this as the velocity of the wind, we now find that our acceleration is (assuming a diameter of 2.04 cm and a γ equal to 0.25 $\mathrm{N}\cdot s^2/\mathrm{m}^4$) 0.02 $\mathrm{m/s}^2$.

The net acceleration is now .06 $\mathrm{m/s}^2$ to the left, as before. Plugging this new value for the difference in the accelerations into equation 3, we find a new compensation distance of 3 cm, off by a hardly noticeable amount. However, if one puts this theory into practice, the bullet arcs dramatically off from our projected course. It misses the target (if the flag is fully flapping) at about twice the breadth of the target's torso, an estimated 60 cm. We can calculate the observed acceleration:

$$a = \frac{2d}{t^2} \tag{6}$$

With t = 1.05 and an estimated 2d of 120, we get an acceleration of 1.09 $\mathrm{m/s}^2$. This corresponds to a wind speed of 1.14 m/s.

4 Conclusion

The error in the observed and the expected results is a stunning 39%. This could be due to any number of factors. First, if we utilized linear, instead of quadratic drag on the bullet, the value we would have gotten would have been much smaller for the compensation distance, due to the nature of the linear drag coefficient. Secondly, there is a big mystery in estimating the appropriate wind speed for the target area, which could range from 1 to several meters per second in magnitude. Thirdly, the programming in the game could have been such that the wind speeds did not obey the laws of classical mechanical physics. The final factor could be the most likely, since changing the parameters of the videogame are as simple as a line of code, also given the entertaining aspect of the game.

To conclude, our results clearly show a discrepancy between the predicted compensation distance of 3 and 120 centimeters when wind resistance along the x axis, normal to the trajectory of the bullet, is taken into account. The Coriolis force, surprisingly, is present and is required to be compensated for at low wind speeds. However, the physics of the wind during this segment of the game do not obey either linear, or quadratic air resistance theory as laid out in classical mechanics.

References

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